The Philippine Statistician, 1995-1996
Vol. 44-45, Nos. 1-8, pp. 93-107

Sampling Audit of Financial Flows and Physical Assets in Bilateral Agreements: Philippines and USAID

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ABSTRACT

Sampling audit using separate and combined ratio estimates was applied on the financial flows and physical assets of Bilateral Agreements between the Philippines and the USAID. This unique procedure made use of recorded or book values and verified or audited accounts.

KEY WORDS: Sampling audit, Recorded or book values, Verified or audited accounts.

1. INTRODUCTION

1.1 The Project

In 1990, the authors served as chief and associate statisticians in a project on the estimation of total financial flows and physical assets generated through ten (10) Bilateral Agreements between the Government of the Philippines (GOP) and the United States Agency for International Development (USAID) for the years 1987-1990. The basic objective is to find out through sampling and other statistical techniques if the total financial flows and physical assets funded by the Agreements have actually been used/received by the local agencies of the GOP at the barangay, municipality or city, and/or the beneficiaries /recipients. These ratio estimation techniques are unique to the types of problems associated with auditing and accounting problems. A group of certified public accountants (CPAs) gathered the data under the administrative control of a reputable Audit/Accounting firm from Metro Manila.[2]

1.2 Statutory Authority: Sampling Audit

As early as August 1964, the United States (U.S.) Congress enacted Public Law 88-521 entitled "An Act to Permit the Use of Statistical Sampling Procedure in the Examination of Vouchers". With this authority, the U.S. General Auditing Office (GAO) started to apply sampling procedures in some initial studies, namely: (a) Statistical Sampling Procedures in Examination of Vouchers (S. Uyeda, 1965); (b) Application of Statistical Sampling Techniques to the Audit of Air Force Disbursing Accounts (H.S. Goldstock, 1966); (c) Use of Statistical Sampling Techniques (USGAO, 1968); and, (d) Acceptance Sampling to Reduce Audit Work (A.E. Burrow, 1976). In the Philippines, there appears no similar authority for its Commission on Audit (COA), GOP.

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[2] SGV Consulting. Briefing Notes and Materials for the Estimation of Cash Disbursements under the USAID Support for Development and other Budget Support Programs. June 1990. Metro Manila.

2. TEN AGREEMENTS AS DOMAINS: STRATIFICATION

The ten (10) Agreements as Domains are coded as to subject matter field and the extent of the coverage (13 regions, GOP). The emphasis is on the Methodology rather than actual data. Some illustrations (Appendix Tables II and III) are given to clarify the methodology. The objective of the entire exercise is the estimation of total financial flow or disbursements for the ten agreements. Several variance reducing techniques such as stratification and the use of the ratio method of estimation were applied to ensure maximum accuracy of the estimates with the least sample size.

2.1 Strata: Implementing Agencies and Agreement.

Under each agreement, there are implementing agencies usually Departments, depending on the nature of the socio-economic program(s). Under each Department are Bureaus, Offices, Institutes, Centers, and/or Regional Offices. Based on the design, these units are either strata or substrata or primary (PSU), secondary (SSU), or tertiary (TSU) sampling units. For the financial flows, the 24 payroll periods, 12 months, 4 quarters are designated as strata or sampling units (SU). At whatever stage, strata or SUs, the optimum design with equal take is applied at each location where the documents/data are filed.

2.2 Location: Where Documents or Book of Accounts are Stored or Maintained

The unique techniques are applied to the documents containing recorded data on disbursements (or financial flows and/or physical assets) in each **location** (accounting/auditing division/section) of a Bureau, Institute, Center, Regional or Provincial Office, or even a Department which serves as Implementing Agencies for each Agreement. For each location, the indicators are arranged in descending order. The ordered list is then divided into three strata (S, M, L) such that the sums of total recorded disbursements are (almost) equal in each stratum. The total number of sample records to be examined are equally allocated in each stratum. Such a design is optimum (Oñate, various years) and the sample sizes used in each stratum is equal to 3 or 4. In addition, with equal take, there is a minimum risk of committing non-sampling errors especially in very large sample surveys like in Sampling Audit. (See Appendix I for List of Indicators).

In each location the sum or total of recorded disbursement are also available. This could be used at any strata or stage levels for the application of ratio estimatior in the estimation of total amount disbursed as verified under the program. High correlation was observed between recorded and verified disbursed amount (0.8-0.99) which indicates that a large gain in precision of the estimates was attained with this method of estimation.

3. SOME BACKGROUND NOTES ON THE FINANCIAL FLOWS AND PHYSICAL ASSETS: AGRARIAN REFORM SUPPORT PROGRAM (ARSP) - AN EXAMPLE

The ARSP is one of 10 agreements and is used as an example or illustration since it contains both Financial and Physical indicators.[3]

[3] SGV Consulting. Briefing Notes and Materials for the Conduct of the Financial/Physical Monitoring of the Agrarian Reform Support Program. August 1990. Metro Manila.

3.1 Primary Engagement Objectives

3.1.1 Financial Flows

To estimate the total amount of disbursements in ARSP eligible budget categories during the eighteen-month period, January 1, 1989 to June 30, 1990, from the Agrarian Reform Fund (ARF) and to whether or not this amount is at least equal to the grant made by the U.S. Government under ARSP. This illustration will give a general idea of the sampling audit procedure applied to the other agreements.

a. A special fund has been set up to provide funding for the Comprehensive Agrarian Reform Program (CARP). This is the Agrarian Reform Fund (ARF) and is accounted for as Fund 158. The ARF is separate and distinct from the General Fund. Contributions of the U.S. Government go into ARF. There are, however, other sources of funding for ARF. Hence the study covered disbursements out of ARF, not specifically expenditures out of the U.S. contributions, since the latter are not separately accounted for.
b. Under Philippine government accounting, expenditures under a budget line are classified into three major expenditure, namely: personnel services (PS), maintenance and other operating expenses (MOOE), and capital outlay (CO). ARSP specifically excludes disbursements for personnel services.

3.1.2 Physical Assets

To estimate the total accomplishment for each of the five physical performance indicators during the period and to ascertain whether or not the following performance parameters were achieved:

Physical Performance Indicators	Performance Benchmark
Areas surveyed (ha.)	182,695
Emancipation patents generated (ha.)	176,987
Number of emancipation patents generated	190,633
Emancipation patents distributed (ha.)	131,223
Number of emancipation patents distributed	138,130

These physical performance indicators relate only to rice and corn lands (PD 27). The period is the eighteen-month period from January 1, 1989 to June 30, 1990.

3.2 The Sampling Frame for the Financial Estimation

The **sampling frame** consists of the universes and sub-universes of all documents containing ARSP eligible disbursements during the eighteen-month period.

a. The sub-domain for which a separate estimate of total eligible disbursements is to be made shall consist of the various locations of an implementing department or agency. A location is defined as any organizational unit of a department which maintains its own separate book of accounts. A location might therefore be any of the following: Head office or department proper; Bureau; Regional office; Provincial office.
b. Two major expenditure categories as financial indicators are eligible under ARSP and these are maintenance and other operating expenses (MOOE) and capital

outlays (CO). There are sub-accounts for each category and these are detailed in Appendix I.

c. To reduce variability for each type of expenditure, a paper stratification by amounts was developed. Three paper strata were applied for each major expenditure type, representing large (L), medium (M) and small (S) amounts.
d. A sample of size n will be randomly drawn from each cell and subjected to review/audit. Each sample observation will have two values, the verified or accounted value and the recorded or book value. The estimation procedures utilized the separate and combined ratio estimators. [4, 5]

3.3 Primary Engagement Objectives

This paper deals primarily with the statistical methodology related to the Primary Engagement Objectives. The ARSP, one of the ten agreements covers both financial flows (disbursements) and physical assets (areas, EPs).

4. SAMPLING DESIGNS AND ESTIMATION PROCEDURES: SEPARATE AND COMBINED RATIOS

4.1 Estimators and Variances

4.1.1 Stratified Sampling (StRS)

Notations

Let

i stratum index,i=1,2,...,L; L=number of strata formed

j observation index in the ith stratum; j=1,2,...,n_i: n_i=number of observations in the ith stratum.

A_{ij} audited/verified amount of the jth observation taken from the ith stratum

R_{ij} recorded or book amount of the jth observation in the ith stratum

Define

${}_AT_i$ total (population) audited amount for the ith stratum

$= \sum_{j=1}^{N_i} A_{ij}$; N_i=population size of the ith stratum

${}_At_i$ unbiased estimator of the total audited amount for the ith stratum $({}_AT_i)$

$$= N_i \left(\frac{\sum_{j=1}^{n_i} A_{ij}}{n_i} \right)$$

4 Oñate, B.T. Ratio Estimation in Multi-Stage Design. Philippine Statistician. Vol. 11, No. 2. June 1962. See also Phil. Statistician Vol. 11, Nos. 3-4, Sept-Dec., 1967.

5 _____ and J.M.O.Bader. 1990. Sampling Surveys and Applications. Oarland Publishers Inc., College, Laguna. Chapter VI.

$\bar{A}_i$ population average audited amount for the ith stratum

$$= \frac{1}{N_i} \sum_{j=1}^{N_i} A_{ij}$$

$\bar{a}_i$ unbiased estimator of the population average audited amount for the ith stratum

$$= \frac{1}{n_i} \sum_{j=1}^{n_i} A_{ij}$$

S_i^2 true variance of the audited amount for the ith stratum

$$= \frac{1}{N_i - 1} \sum_{j=1}^{N_i} (A_{ij} - \bar{A}_i)^2$$

s_i^2 sample variance (unbiased estimator) of the audited amount for the ith stratum

$$= \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (A_{ij} - \bar{a}_i)^2$$

The unbiased estimate of the total audited amount denoted by , ${}_A t_{ST}$, is computed as

$$ {}_A t_{ST} = \sum_{i=1}^{L} {}_A t_i \tag{1}$$

Variances and variance estimates

Under stratified random sampling, the true variance of the estimate of the total audited amount for the ith stratum (${}_A t_i$) is defined as

$$\text{Var}({}_A t_i) = \frac{N_i (N_i - n_i)}{n_i} S_i^2 \tag{2}$$

This is estimated (unbiasedly) by

$$\text{var}({}_A t_i) = \frac{N_i (N_i - n_i)}{n_i} s_i^2 \tag{3}$$

The true variance of the estimate total audited amount is defined as

$$\text{Var}({}_A t_{ST}) = \sum_{i=1}^{L} \text{Var}({}_A t_i) \tag{4}$$

This is estimated by

$$\text{var}({}_A t_{ST}) = \sum_{i=1}^{L} \text{var}({}_A t_{ST}) \tag{5}$$

The estimation procedures described is applied in the component or stages in the estimation of totals in E_1(PS), E_2(MOOE), and E_3(CO) for agencies/regions in the

departments which are included in each agreement. The details for the separate and combined ratios are given in the illustration.

4.1.2 Personnel Expenditures (E_1): Two-stage design

A two-stage design for personnel expenditures will reduce the workload involved in capturing transaction data. Copying of all payroll lines will be limited to the payrolls picked as samples from a total of 24 payrolls during the year. Complete frame data are obtained for only the three PSU sampled payrolls.

The estimator of the population total of verified or audited disbursements of E_1 during the year (T_A) is:

$$t_A^* = \frac{N}{n}\sum_{j=1}^{n} M_j \sum_{k=1}^{m_j} \frac{A_{jk}}{m_j} = \frac{N}{n}\sum_{j=1}^{n} M_j \bar{a}_j \qquad (6)$$

where

- N is the number of primary sampling units or PSUs in the ith stratum. The PSUs refer to the pay periods during the year. In general, N=24.
- n is the number of sample PSUs and is set to equal 3.
- M_j is the number of secondary sampling units or SSUs (transactions or lines in the payroll) in the jth PSU and may differ from PSU to PSU.
- m_j is the number of sample SSUs and this is set at 4 for all PSUs
- $\bar{a}_j$ is the sample mean of the audited values of the SSUs taken from the jth PSU.

The estimator of the variance t_A^* is:

$$\text{var}(t_A^*) = \frac{N}{n}\sum_{j=1}^{n} \frac{M_j(M_j - m_j)}{m_j} s_j^2 + \frac{N(N-n)}{n}\sum_{j=1}^{n} \frac{(M_j \bar{a}_j - \bar{a})^2}{n-1} \qquad (7)$$

where

- s_j^2 sample variance of the audited amounts.
$$= \sum_{k=1}^{m_j} \frac{(A_{jk} - \bar{a}_j)^2}{m_j - 1}$$
- $\bar{a}$ sample mean of estimated total audited amounts between PSU
$$= \sum_{j=1}^{n} \frac{M_j \bar{a}_j}{n}$$

It must be noted that (6) consists of two components, namely: (i) the first is the between SSU or within PSU variance; and, (ii) the second is the between PSU variance which usually accounts for a larger portion of the total variance.

These formulas are applied separately for each size stratum (S, M, L). These estimation procedures are applied to the R_{ij}s to obtain similar estimators for the recorded values. (See Sec. 4.2 and Appendix II).

4.1.3 Separate Ratio Estimator

In each stratum, a separate ratio estimator of the total amount disbursed as verified is defined as

$$ {}_{Q}t_i = \left(\frac{{}_{A}t_i}{{}_{R}t_i} \right) {}_{R}T_i \tag{8} $$

and an estimate of its variance is defined as

$$ \text{var}({}_{Q}t_i) = \left(\frac{{}_{A}t_i}{{}_{R}t_i} \right)^2 \left\{ \text{var}({}_{A}t_i) + \text{var}({}_{R}t_i) - 2\,\text{cov}({}_{A}t_i, {}_{R}t_i) \right\} \tag{9} $$

where

$$ \text{cov}({}_{A}t_i, {}_{R}t_i) = \frac{N_i(N_i - n_i)}{n_i} \sum_{j=1}^{n_i} \frac{(A_{ij} - \bar{a}_i)(R_{ij} - \bar{r}_i)}{n_i - 1} \tag{10} $$

Thus, for each expenditure item, the estimated total amount disbursed as verified/audited is

$$ {}_{Q}t = \sum_{i=1}^{L} {}_{Q}t_i \tag{11} $$

The estimate of the variance of (11) is defined as

$$ \text{var}({}_{Q}t) = \sum_{i=1}^{L} \text{var}({}_{Q}t_i) \tag{12} $$

4.1.4 Combined Ratio Estimator

The combined ratio estimator of the total amount disbursed as verified for each major expenditure item is defined as

$$ \tilde{t}_c = \left(\frac{{}_{A}t_{ST}}{{}_{R}t_{ST}} \right) {}_{R}T \tag{13} $$

The estimated variance of (13) is defined as

$$\text{var}(\tilde{t}_C) = \text{var}({}_A t_{ST}) + \left(\frac{{}_A t_{ST}}{{}_R t_{ST}}\right)^2 \text{var}({}_R t_{ST}) - 2\left(\frac{{}_A t_{ST}}{{}_R t_{ST}}\right)\text{cov}({}_A t_{ST}, {}_R t_{ST}) \qquad (14)$$

These formulas are applied to data on the A_{ij} and R_{ij}. Details are given in the illustration (Sec. 4.2 and Appendix II).

4.1.5 Ratio Estimation in a Two-stage Design

In estimating the total amount disbursed under personnel services as verified, a ratio-type estimator was used to take advantage of the high degree of linear relationship between total amount audited and reported which would lead to improved precision. Since a two-stage design was used in estimating total amount disbursed as audited for (PS), the following estimator was used;

$${}_Q t^* = \left(\frac{t_A^*}{t_R^*}\right)_R T \qquad (15)$$

The estimated variance of (14) is defined as

$$\text{var}({}_Q t^*) = \text{var}(t_A^*) + \left(\frac{t_A^*}{t_R^*}\right)^2 \text{var}(t_R^*) - 2\left(\frac{t_A^*}{t_R^*}\right)\text{cov}(t_A^*, t_R^*) \qquad (16)$$

4.1.6 Optimum Allocation with Equal Take

Ignore the PSUs (payroll period) and array the recorded data (R_{ij}) into S, M, and L strata eliminating very large PSUs (say 16-29 February 1989 payroll for complete enumeration). The two-stage design will be replaced by a simple stratified separate and combined ratio estimators where instead of N=23 and n=3, there will be 3 strata, namely: S, M, and L. Within each stratum

$$n_i^* = n/3 = 3 \text{ or } 4 \qquad (17)$$

and

$$\frac{N_i S_i}{\sum_{i=1}^{3} N_i S_i} = \frac{1}{3}$$

where N_i total number of recorded transactions or payrolls within the ith stratum (i=1,2,3)

S_i standard deviation of the R_{ij}s within the ith stratum.

There is no longer any PSUs or SSUs. This is a more efficient design but will need more labor for the ordering of the data set into the S, M, and L strata. The two-stage design was used since payroll periods are more or less stable except when bonuses, 13th month pay and other incentives are given resulting to larger payrolls. (See Sec. 4.1, 2a).

4.1.7 Ratio Estimation in a Three-stage Sampling Design

The results presented in Section 5 can be easily extended to a three-stage design. With this design, the variance estimates will consist of three components, namely: (a) Between PSUs; (b) Between SSUs; and, (c) Between tertiary sampling units (TSUs) variability. It is expected that the contribution of the between PSU variance will be large. Only the PSU variance will be used in the estimation of variances which may later be adjusted if found not so in the experiments.

4.1.8 Confidence Interval Estimates

At each grant agreement, the estimates ${}_Qt^*$ and its variance $\text{var}({}_Qt^*)$, for each implementing office for a particular agreement will be added for all offices (locations), resulting in estimates ${}_Qt^{**}$ and $\text{var}({}_Qt^{**})$ for the grant agreement.

A 95% confidence interval estimate of the total amount disbursed for each grant agreement as audited will be:

$$\text{Upper Limit:} \quad {}_Qt^{**} + z_{0.025}\sqrt{\text{var}({}_Qt^{**})}$$

$$\text{Lower Limit:} \quad {}_Qt^{**} - z_{0.025}\sqrt{\text{var}({}_Qt^{**})}$$

where $z_{0.025} = 1.96$

At the level of all ten agreements combined, the estimates for the total valid disbursements and the variance will be generated following the same manner as described above. Similarly, a 95% confidence interval estimate will be constructed.

4.2 Illustrations

The methodology described in 4.1 is applied to specific data for the three types of expenditure items, namely: E_1 - Personnel Services (PS); E_2 - Maintenance and Other Operating Expenses (MOOE); and, E_3 - Capital Outlay (CO). The number of strata may range from 3 (E_1) to 4 (E_2 and E_3) where the size of the samples within each stratum is equal to 4 (optimum with equal take).

The MOOE data were obtained from the Agricultural Training Institute (ATI) for year ended 1988 (Appendix II-1). The 2,851 items were arranged in descending order and the 10 largest items were verified and audited to total ₱ 7,416,957 (complete enumeration, CE). The four remaining strata were almost of equal weight and 4 items (sample) were drawn from each stratum using a random systematic start. Thus, L=4 and $n_i = n^* = 4$. The data shown in Appendix II-1 and the methodology of (3.3) and (3.4) were used and details are given in Appendix II-2. Including the CE (verified), the estimates are:

Separate Ratio	₱ 31.382 million	(rounded)
Combined Ratio	₱ 31.370 million	(rounded)
Difference	**₱ 0.012 million**	(rounded)

Also, the separate stratum ratios ranged from 0.975 to 0.999 while the combined ratio is 0.998. The CVs of such estimates are generally less than 1%.

If these estimates are pooled by sub-domains and domains, then the CVs of the estimates are less than 0.5%. In addition, each estimate (separate or combined) exceeds the targets for each agreement.

Another example is given in Appendix III. Three PSUs were drawn (n=3) where each PSU represents a payroll (E_1) period of 2 weeks and the total number of 2-week period is N=24. In each PSU, the payrolls were arranged into three strata, namely: Small (S), Medium (M) and Large (L) payrolls to obtain both verified or audited (A_{ij}) and the recorded (R_{ij}) amounts. Note that the total recorded amount (R_{ij}) for each stratum, each PSU (R..), each of the N payroll periods are available for adjustment. These are the unique dimensions on the use of either the separate and combined ratio in stratified multi-stage design. Actually, one payroll was enumerated due to its size such that N=23 and n=3 (PSUs).

APPENDIX 1

Financial (F_{ij}) and Physical (areas and EPs) Indicators

A. FINANCIAL FLOWS (DISBURSEMENT INDICATORS), $F_{(ij)}$

Sub-Accounts and Codes for Maintenance and Other Operating Expenses and Capital Outlays

Sub-Accounts	Codes	Indicators (F_{ij})
200		Maintenance and Other Operating Expenses
	00	Miscellaneous Maintenance and Other Operating Expenses
	02	Traveling Expenses
	03	Communication Expenses
	04	Repair and Maintenance of Government Facilities
	05	Transportation Expenses
	06	Other Services
	07	Supplies and Materials
	08	Rents
	09	Interests
	10	Grants, Subsidies and Contributions
	11	Awards and Indemnities
	12	Loan Repayments and Sinking Fund Contribution
	13	Loses/Depreciation/Depletion
	14	Water, Illumination and Power Services
	15	Social Security Benefits, Rewards and other Claims
	16	Auditing Services
	17	Maintenance of Motor Vehicles used for official travels of officials and employees
	18	Discretionary Expenses
	19	Representation Expenses
	20	Extraordinary/Contingency/Emergency Expenses
	21	Taxes, Duties and Fees
	22	Trading/Production
300		Capital Outlay
	31	Land and Land Improvement Outlay
	32	Buildings and Structures Outlay
	33	Furniture, Fixtures, Equipment, Work Animals and Books Outlay
	34	Investments Outlay
	35	Loans Outlay

B. PHYSICAL PERFORMANCE INDICATORS

Areas Surveyed in Hectares	$_sA_h$
Emancipation Patents Generated in Hectares	$_gP_h$
Number of Emancipation Patents Generated	$_gP_n$
Emancipation Patents Distributed in Hectares	$_dP_h$
Number of Emancipation Patents Distributed	$_dP_n$

APPENDIX 2

Illustration 1

Agricultural Training Institute
Schedule of Sampled Maintenance and Other Operating Expenses (MOOE)
For the Year Ended December 31, 1988

	VERIFIED	RECORDED	VERIFIED	RECORDED	VERIFIED	RECORDED	VERIFIED	RECORDED
	269,361	269,361						
	161,123	161,303						
	100,241	100,641						
	84,600	84,600						
			67,809	67,809				
			55,400	55,500				
			50,000	50,000				
			43,127	43,127				
					25,350	25,350		
					8,000	8,000		
					4,150	4,150		
					2,613	2,613		
							1,500	1,500
							870	896
							520	565
							280	280
TOTAL	615,325	615,905	216,336	216,336	40,113	40,313	3,170	3,251
N_i	50		136		900		1,755	

APPENDIX 2.2

A. STRATIFIED RANDOM SAMPLING WITH SEPARATE AND COMBINED RATIOS

$_At_1 = N_1\bar{a}_1 = (50)(615{,}325/4) = 7{,}691{,}562$

$_Rt_1 = N_1\bar{r}_1 = (50)(615{,}905/4) = 7{,}698{,}812$

$_At_2 = N_2\bar{a}_2 = (136)(216{,}336/4) = 7{,}355{,}424$

$_At_2 = N_2\bar{r}_2 = (136)(216{,}456/4) = 7{,}359{,}504$

$_At_3 = N_3\bar{a}_3 = (900)(40{,}113/4) = 9{,}025{,}425$

$_Rt_3 = N_3\bar{r}_3 = (900)(40{,}313/4) = 9{,}070{,}425$

$_At_4 = N_4\bar{a}_4 = (1{,}755)(3{,}170/4) = 1{,}390{,}837$

$_Rt_4 = N_4\bar{r}_4 = (1{,}755)(3{,}251/4) = 1{,}426{,}376$

$q_1 = {}_At_1/{}_Rt_1 = 0.999$

$q_2 = {}_At_2/{}_Rt_2 = 0.999$

$q_3 = {}_At_3/{}_Rt_3 = 0.995$

$q_4 = {}_At_4/{}_Rt_4 = 0.975$

$_{q1}t_A^* = (q_1)({}_RT_1) = (0.999)(7{,}537{,}077) = 7{,}529{,}979$

$_{q2}t_A^* = (q_2)({}_RT_2) = (0.999)(7{,}547{,}902) = 7{,}543{,}718$

$_{q3}t_A^* = (q_3)({}_RT_3) = (0.995)(7{,}545{,}017) = 7{,}507{,}585$

$_{q4}t_A^* = (q_4)({}_RT_4) = (0.975)(1{,}419{,}081) = 1{,}383{,}724$

B. SEPARATE RATIO

$${}_At_{ST}^* = {}_{q1}t_A^* + {}_{q2}t_A^* + {}_{q3}t_A^* + {}_{q4}t_A^* + CE_A = 23{,}965{,}006 + 7{,}416{,}957 = 31{,}381{,}963$$

C. COMBINED RATIO

$${}_At_{ST} = {}_At_1 + {}_At_2 + {}_At_3 + {}_At_4 = 7{,}691{,}562 + 7{,}355{,}424 + 9{,}025{,}425 + 1{,}390{,}837 = 25{,}463{,}248$$

$${}_Rt_{ST} = {}_Rt_1 + {}_Rt_2 + {}_Rt_3 + {}_Rt_4 = 7{,}698{,}812 + 7{,}359{,}504 + 9{,}070{,}425 + 1{,}426{,}376 = 25{,}555{,}117$$

$$t_A^* = ({}_At_{ST}/{}_Rt_{ST})({}_RT) + CE_A = (0.996)(24{,}049{,}077) + 7{,}416{,}957 = 31{,}369{,}837$$

NOTE: CE_A - Complete Enumeration

APPENDIX 3

Illustration 2

Agricultural Training Institute
Schedule of Sampled Disbursements for Personnel Services (E_1)
For the Payroll Period March 1 to 15, 1988 ($PP_1 = PSU_1$)

	STRATUM 1 (L)		STRATUM 2 (M)		STRATUM 3 (S)	
ITEM	VERIFIED	RECORDED	VERIFIED	RECORDED	VERIFIED	RECORDED
VERIFIER	(A)	(R)	(A)	(R)	(A)	(R)
21-4	1,290.00	1,294.00				
34-4	875.00	875.00				
51-24	795.00	795.00				
24-12	740.00	740.00				
51-19			650.00	650.00		
20-23			600.00	620.00		
4-12			575.00	584.00		
9-4			552.00	552.00		
44-16					550.00	550.00
29-16					370.00	370.00
27-11					200.00	220.00
29-10					160.00	160.00
TOTAL	3,700.00	3,704.00	2,377.00	2,406.00	1,280.00	1,300.00
MEAN	925.00	926.00	594.25	601.50	320.00	325.00
N_i	181	181	271	271	556	556
${}_At_i, {}_Rt_i$	167,425.00	167,606.00	161,041.75	163,006.50	177,920.00	180,700.00
q_i (ratio)	0.999		0.988		0.985	
${}_RT_i$		165,874.00		165,964.00		165,783.00
${}_qt_i$	165,694.87		163,963.60		163,232.49	

PSU_1 (PP_1) Estimated Total = $\sum_{i=1}^{3} {}_q t_i = 492{,}890.97$

Combined Ratio (at PSU_1 level) = $\left(\sum_{i=1}^{3} {}_A t_i \,/\, \sum_{i=1}^{3} {}_R t_i\right) = (506{,}386.75 / 511{,}312.50) = 0.990$

Combined Ratio Estimated Total = $q_{ST_1}\left(\sum_{i=1}^{3} {}_R T_i\right) = (0.990)(497{,}621.00) = 492{,}827.15$

Agricultural Training Institute
Schedule of Sampled Disbursements for Personnel Services (E_1)
For the Payroll Period July 1 to 15, 1988 ($PP_2 = PSU_2$)

	STRATUM 1 (L)		STRATUM 2 (M)		STRATUM 3 (S)	
ITEM	VERIFIED	RECORDED	VERIFIED	RECORDED	VERIFIED	RECORDED
VERIFIER	(A)	(R)	(A)	(R)	(A)	(R)
1-6	1,700.00	1,700.00				
20-16	872.00	892.00				
52-24	790.00	794.00				
7-11	739.00	739.00				
13-18			630.00	638.00		
55-16			620.00	620.00		
56-22			565.00	570.00		
44-19			554.00	554.00		
35-21					540.00	540.00
35-21					340.00	350.00
27-20					220.00	220.00
54-13					125.00	130.00
TOTAL	4,101.00	4,125.00	2,369.00	2,382.00	1,225.00	1,240.00
MEAN	1,025.25	1,031.25	592.25	595.50	306.25	310.00
N_i	175	175	262	262	534	534
${}_At_i, {}_Rt_i$	179,418.75	180,468.75	155,169.50	156,021.00	163,537.50	165,540.00
q_i (ratio)	0.994		0.995		0.988	
${}_RT_i$		165,874.00		165,964.00		165,783.00
${}_qt_i$	164,908.91		165,058.24		163,777.56	

PSU_2 (PP_2) Estimated Total = $\sum_{i=1}^{3} {}_q t_i = 493,744.70$

Combined Ratio (at PSU_2 level) = $\left(\sum_{i=1}^{3} {}_A t_i \,/\, \sum_{i=1}^{3} {}_R t_i \right) = (498,125.75 / 502,029.75) = 0.992$

Combined Ratio Estimated Total = $q_{ST_2} \left(\sum_{i=1}^{3} {}_R T_i \right) = (0.992)(497,621.00) = 493,751.20$